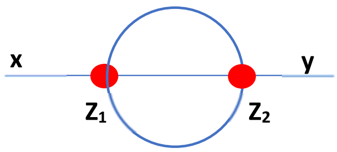
RODOLFO GUIDOBONO	**CURSO TEORÍA CUÁNTICA DE CAMPOS** **EJERCICIO CAPÍTULO 81**	**JAVIER GARCÍA**

EJERCICIO (31:15)

Calcular el diagrama de Feynman, en el espacio de momentos, de:



Siguiendo las reglas de Feynman:

1. Por cada línea, un término:

   $$\frac{i}{{p_i}^2 - m^2 + i\,\varepsilon}$$

   Siendo p_1 el momento de "x" a z_1 y el momento p_2 de "y" a z_2 (por el momento consideramos esta dirección como positiva), queda:

   $$\frac{i}{{p_1}^2 - m^2 + i\,\varepsilon}\,\frac{i}{{p_2}^2 - m^2 + i\,\varepsilon}$$

2. Por cada vértice, un término (-i λ), correspondiendo, en este caso:

   $$(-i\lambda)^2$$

3. Dividir por el factor de simetría; en este caso igual a 6
4. Poner un delta de Dirac que asegure la conservación del movimiento en cada nodo (positivo si entra, negativo si sale)

   $$(2\,\pi)^4 \delta^{(4)}{}_{(p_1 - k_1 - k_2 - k_3)}\,(2\,\pi)^4 \delta^{(4)}{}_{(k_1 + k_2 + k_3 - p_2)}$$

5. Integrar por todos los momentos internos, no restringidos, que en este caso son 3 y que denominamos k_1, k_2 y k_3:

   $$\int \frac{d^4k_1}{(2\,\pi)^4}\left(\frac{i}{{k_1}^2 - m^2 + i\,\varepsilon}\right)\int \frac{d^4k_2}{(2\,\pi)^4}\left(\frac{i}{{k_2}^2 - m^2 + i\,\varepsilon}\right)\int \frac{d^4k_3}{(2\,\pi)^4}\left(\frac{i}{{k_3}^2 - m^2 + i\,\varepsilon}\right)$$

Resulta:

$$\boxed{\begin{aligned} \boldsymbol{G_{(p,q)}} &= \boldsymbol{\frac{1}{6}(-i\lambda)^2 \frac{i}{{p_1}^2 - m^2 + i\,\varepsilon}\,\frac{i}{{p_2}^2 - m^2 + i\,\varepsilon}} \\ &\boldsymbol{\int \frac{d^4k_1}{(2\,\pi)^4}\left(\frac{i}{{k_1}^2 - m^2 + i\,\varepsilon}\right)\int \frac{d^4k_2}{(2\,\pi)^4}\left(\frac{i}{{k_2}^2 - m^2 + i\,\varepsilon}\right)\int \frac{d^4k_3}{(2\,\pi)^4}\left(\frac{i}{{k_3}^2 - m^2 + i\,\varepsilon}\right)} \\ &\boldsymbol{(2\,\pi)^4 \delta^{(4)}{}_{(p_1 - k_1 - k_2 - k_3)}\,(2\,\pi)^4 \delta^{(4)}{}_{(k_1 + k_2 + k_3 - p_2)}} \end{aligned}}$$

Para llegar al mismo resultado debemos calcular la transformada de Fourier de la representación del espacio de posiciones:

$$G_{(x,y)} = \frac{1}{6}(-i\lambda)^2 \int d^4z_1 \int d^4z_2 \;\; \Delta_F(x-z_1)\, \Delta_F(z_1-z_2)\, \Delta_F(z_1-z_2)\, \Delta_F(z_1-z_2)\, \Delta_F(y-z_2)$$

Donde:

$$\Delta_F(x-y) = i \int \frac{d^4k}{(2\,\pi)^4} \frac{e^{-i\,k\,(x-y)}}{k^2 - m^2 + i\,\varepsilon}$$

Resultando para nuestro diagrama:

$$\Delta_F(x-z_1) = \int \frac{d^4p_1}{(2\,\pi)^4} \frac{i}{{p_1}^2 - m^2 + i\,\varepsilon} e^{-i\,p_1\,(x-z_1)}$$

$$\Delta_F(z_1-z_2) = \int \frac{d^4k_1}{(2\,\pi)^4} \frac{i}{{k_1}^2 - m^2 + i\,\varepsilon} e^{-i\,k_1\,(z_1-z_2)}$$

$$\Delta_F(z_1-z_2) = \int \frac{d^4k_2}{(2\,\pi)^4} \frac{i}{{k_2}^2 - m^2 + i\,\varepsilon} e^{-i\,k_2\,(z_1-z_2)}$$

$$\Delta_F(z_1-z_2) = \int \frac{d^4k_3}{(2\,\pi)^4} \frac{i}{{k_3}^2 - m^2 + i\,\varepsilon} e^{-i\,k_3\,(z_1-z_2)}$$

$$\Delta_F(y-z_2) = \int \frac{d^4p_2}{(2\,\pi)^4} \frac{i}{{p_2}^2 - m^2 + i\,\varepsilon} e^{-i\,p_2\,(y-z_2)}$$

$$G_{(x,y)} = \frac{1}{6}(-i\lambda)^2 \int d^4z_1 \int d^4z_2 \int \frac{d^4p_1}{(2\,\pi)^4} \frac{i}{{p_1}^2 - m^2 + i\,\varepsilon} e^{-i\,p_1\,(x-z_1)}$$
$$\int \frac{d^4k_1}{(2\,\pi)^4} \frac{i}{{k_1}^2 - m^2 + i\,\varepsilon} e^{-i\,k_1\,(z_1-z_2)} \int \frac{d^4k_2}{(2\,\pi)^4} \frac{i}{{k_2}^2 - m^2 + i\,\varepsilon} e^{-i\,k_2\,(z_1-z_2)}$$
$$\int \frac{d^4k_3}{(2\,\pi)^4} \frac{i}{{k_3}^2 - m^2 + i\,\varepsilon} e^{-i\,k_3\,(z_1-z_2)} \int \frac{d^4p_2}{(2\,\pi)^4} \frac{i}{{p_2}^2 - m^2 + i\,\varepsilon} e^{-i\,p_2\,(y-z_2)}$$

Reordenando para separar z_1 y z_2:

$$G_{(x,y)} = \frac{1}{6}(-i\lambda)^2 \int d^4z_1 \int d^4z_2 \int \frac{d^4p_1}{(2\,\pi)^4} \int \frac{d^4k_1}{(2\,\pi)^4} \int \frac{d^4k_2}{(2\,\pi)^4} \int \frac{d^4k_3}{(2\,\pi)^4} \int \frac{d^4p_2}{(2\,\pi)^4}$$
$$\frac{i}{{p_1}^2 - m^2 + i\,\varepsilon} \frac{i}{{k_1}^2 - m^2 + i\,\varepsilon} \frac{i}{{k_2}^2 - m^2 + i\,\varepsilon} \frac{i}{{k_3}^2 - m^2 + i\,\varepsilon} \frac{i}{{p_2}^2 - m^2 + i\,\varepsilon}$$
$$e^{-i\,x\,(p_1)}\; e^{-i\,y\,(p_2)}\; e^{-i\,z_1\,(-p_1+k_1+k_2+k_3)}\; e^{-i\,z_2\,(-p_2-k_1-k_2-k_3)}$$

$$G_{(x,y)} = \frac{1}{6}(-i\lambda)^2 \int \frac{d^4p_1}{(2\,\pi)^4}\int \frac{d^4k_1}{(2\,\pi)^4}\int \frac{d^4k_2}{(2\,\pi)^4}\int \frac{d^4k_3}{(2\,\pi)^4}\int \frac{d^4p_2}{(2\,\pi)^4}$$

$$\frac{i}{{p_1}^2-m^2+i\,\varepsilon}\frac{i}{{k_1}^2-m^2+i\,\varepsilon}\frac{i}{{k_2}^2-m^2+i\,\varepsilon}\frac{i}{{k_3}^2-m^2+i\,\varepsilon}\frac{i}{{p_2}^2-m^2+i\,\varepsilon}$$

$$e^{-i\,x\,(p_1)}\,e^{-i\,y\,(p_2)}\int d^4z_1\;e^{-i\,z_1\,(-p_1+k_1+k_2+k_3)}\int d^4z_2\;e^{-i\,z_2\,(-p_2-k_1-k_2-k_3)}$$

$$G_{(x,y)} = \frac{1}{6}(-i\lambda)^2 \int \frac{d^4p_1}{(2\,\pi)^4}\int \frac{d^4k_1}{(2\,\pi)^4}\int \frac{d^4k_2}{(2\,\pi)^4}\int \frac{d^4k_3}{(2\,\pi)^4}\int \frac{d^4p_2}{(2\,\pi)^4}$$

$$\frac{i}{{p_1}^2-m^2+i\,\varepsilon}\frac{i}{{k_1}^2-m^2+i\,\varepsilon}\frac{i}{{k_2}^2-m^2+i\,\varepsilon}\frac{i}{{k_3}^2-m^2+i\,\varepsilon}\frac{i}{{p_2}^2-m^2+i\,\varepsilon}$$

$$e^{-i\,x\,(p_1)}\,e^{-i\,y\,(p_2)}\int d^4z_1\;e^{i\,z_1\,(p_1-k_1-k_2-k_3)}\int d^4z_2\;e^{i\,z_2\,(p_2+k_1+k_2+k_3)}$$

Como:

$$\int \boldsymbol{d^4x\,e^{i\,(a_1-a_2)x} = (2\,\pi)^4\delta^{(4)}{}_{(a_1-a_2)}}$$

Entonces:

$$G_{(x,y)} = \frac{1}{6}(-i\lambda)^2 \int \frac{d^4p_1}{(2\,\pi)^4}\int \frac{d^4k_1}{(2\,\pi)^4}\int \frac{d^4k_2}{(2\,\pi)^4}\int \frac{d^4k_3}{(2\,\pi)^4}\int \frac{d^4p_2}{(2\,\pi)^4}$$

$$\frac{i}{{p_1}^2-m^2+i\,\varepsilon}\frac{i}{{k_1}^2-m^2+i\,\varepsilon}\frac{i}{{k_2}^2-m^2+i\,\varepsilon}\frac{i}{{k_3}^2-m^2+i\,\varepsilon}\frac{i}{{p_2}^2-m^2+i\,\varepsilon}$$

$$e^{-i\,x\,p_1}\,e^{-i\,y\,p_2}(2\,\pi)^4\delta^{(4)}{}_{(p_1-k_1-k_2-k_3)}\,(2\,\pi)^4\delta^{(4)}{}_{(p_2+k_1+k_2+k_3)}$$

Cambiando el sentido de p_2 desde z_2 hacia y queda:

$$G_{(x,y)} = \frac{1}{6}(-i\lambda)^2 \int \frac{d^4p_1}{(2\,\pi)^4}\int \frac{d^4k_1}{(2\,\pi)^4}\int \frac{d^4k_2}{(2\,\pi)^4}\int \frac{d^4k_3}{(2\,\pi)^4}\int \frac{d^4p_2}{(2\,\pi)^4}$$

$$\frac{i}{{p_1}^2-m^2+i\,\varepsilon}\frac{i}{{k_1}^2-m^2+i\,\varepsilon}\frac{i}{{k_2}^2-m^2+i\,\varepsilon}\frac{i}{{k_3}^2-m^2+i\,\varepsilon}\frac{i}{{p_2}^2-m^2+i\,\varepsilon}$$

$$e^{-i\,x\,p_1}\,e^{-i\,y\,p_2}(2\,\pi)^4\delta^{(4)}{}_{(p_1-k_1-k_2-k_3)}\,(2\,\pi)^4\delta^{(4)}{}_{(k_1+k_2+k_3-p_2)}$$

$$G_{(x,y)} = \int \frac{d^4p_1}{(2\,\pi)^4}\int \frac{d^4p_2}{(2\,\pi)^4}e^{-i\,x\,p_1}\,e^{-i\,y\,p_2}\frac{1}{6}(-i\lambda)^2\int \frac{d^4k_1}{(2\,\pi)^4}\int \frac{d^4k_2}{(2\,\pi)^4}\int \frac{d^4k_3}{(2\,\pi)^4}$$

$$\frac{i}{{p_1}^2-m^2+i\,\varepsilon}\frac{i}{{k_1}^2-m^2+i\,\varepsilon}\frac{i}{{k_2}^2-m^2+i\,\varepsilon}\frac{i}{{k_3}^2-m^2+i\,\varepsilon}\frac{i}{{p_2}^2-m^2+i\,\varepsilon}$$

$$(2\,\pi)^4\delta^{(4)}{}_{(p_1-k_1-k_2-k_3)}\,(2\,\pi)^4\delta^{(4)}{}_{(k_1+k_2+k_3-p_2)}$$

La transformada de Fourier de G(x,y) es:

$$\widetilde{G_{(x,y)}} = \frac{1}{6}(-i\lambda)^2 \frac{i}{{p_1}^2 - m^2 + i\,\varepsilon} \frac{i}{{p_2}^2 - m^2 + i\,\varepsilon} \int \frac{d^4k_1}{(2\,\pi)^4} \int \frac{d^4k_2}{(2\,\pi)^4} \int \frac{d^4k_3}{(2\,\pi)^4} \frac{i}{{k_1}^2 - m^2 + i\,\varepsilon} \frac{i}{{k_2}^2 - m^2 + i\,\varepsilon} \frac{i}{{k_3}^2 - m^2 + i\,\varepsilon}\,(2\,\pi)^4 \delta^{(4)}{}_{(p_1-k_1-k_2-k_3)}\,(2\,\pi)^4 \delta^{(4)}{}_{(k_1+k_2+k_3-p_2)}$$

Hemos llegado al mismo resultado 😊